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from: P. R. Schunk, 9111, MS0826

subject: POROUS MEDIA CAPABILITIES/TUTORIAL FOR GOMA. USER GUIDANCE FOR SATURATED POROUS PENETRATION PROBLEMS (GT-008.3)

keywords: porous media, Darcy's law, porous flow, porous deformable substrate, impregnation, knife coating

input records: Media Type=POROUS=BRINKMAN, Media Type=CONTINUOUS, DARCY_CONTINUOUS, POROUS_PRESSURE, VELO_TANGENT_SOLID, POROUS_KIN, VP_EQUIL, VN_POROUS, POROUS_CONV, POROUS_FLUX, POROUS_PRESSURE

Introduction

This tutorial assumes the user has gone through the beginner's training tutorial on GOMA and SEAMS. If you would like copies of that tutorial, contact Duane Labreche (dalabre@sandia.gov), or search the secure web site.

GOMA has extensive capability for modeling transport phenomena in porous media, including several formulations of the porous transport equations, saturated and unsaturated flow, and even poroelastic stress constitutive equations using the effective stress principle. Three reports are important to the discussion here: Gartling et al. (1996), Schunk et al. (1998) and Cairncross et al. (1996). The first report gives a comprehensive overview and detailed study of the computational issues surrounding coupled fluid and porous flow problems. That report details the comparison between a Darcy formulation and a Brinkman formulation for the porous material, and the implications of each formulation to the interfacial boundary conditions between porous and continuous phases. Those boundary conditions are central to this tutorial. The second report describes most of the boundary conditions and the governing equations available in GOMA 2.0 for modeling transport in porous media. However, because that report is a user's manual for GOMA, the information is spread out and not detailed--and as it turns out not in all cases complete; that is being corrected now (11/98). The third report by Cairncross et al. (1998) describes in detail the equations, boundary conditions, constitutive relations, and even the necessary GOMA input for a partially-saturated poroelastic sol-gel dip coating process. That report is extremely comprehensive with regard to that problem, but has little or no information regarding other situations often encountered in manufacturing and technology involving porous media.

Problems involving fluid and species transport in porous media abound in manufacturing. Two important applications with respect to GOMA involve porous penetration in continuous liquid film coating and metals joining processes; in these and other cases a porous medium is imbibing or expelling fluid from an adjacent layer (depending on the external penetration pressure or internal elastic network stress). In this memo we focus on problems of this class and how they are modeled with GOMA. Before that, however, we attempt to consolidate in one place all equations and boundary conditions that one can deploy with GOMA for modeling transport coupled fluid flow and porous flow problems.

MATERIAL COVERED IN THIS DOCUMENT:

*Files for these problems are contained in a tar file called **porous_template.tar.gz**. The two subdirectories are **general_plug** and **knife**. The tar file can be found on the secure web site.*

Current Capability for Modeling Coupled Fluid Flow and Porous Flow Problems with GOMA

We begin by reviewing the relevant governing differential equations for bulk materials in coupled porous penetration problems. Penetration problems usually involve a continuous fluid medium. In that medium, we can state mathematically the conservation of momentum balance, with viscous, inertial, gravitational, and pressure forces, as:

$$\rho\frac{d\boldsymbol{v}}{dt} = -\rho(\boldsymbol{v} - \boldsymbol{v}_m) \bullet \nabla\boldsymbol{v} + \nabla\bullet\boldsymbol{T} + \boldsymbol{g} \quad (1)$$

$\boldsymbol{v}$ is the mass-averaged velocity of the fluid, ρ is the density of the fluid, $\boldsymbol{T}$ is the stress tensor, and $\boldsymbol{g}$ is the body force acting on the fluid. $\boldsymbol{v}_m$ is the velocity of the mesh, which is zero for steady state problems.

Typically Eq. 1 requires some sort of equation-of-state or conservation-of-mass law so that the pressure in the total stress T can be determined. For the discussion in this memo, we consider the fluid to be incompressible and hence the velocity field to be solenoidal, viz.

$$\nabla\bullet\boldsymbol{v} = 0 \quad (2)$$

The stress tensor in Newtonian or generalized-Newtonian fluids is proportional to the rate-of-strain tensor, $\boldsymbol{D}$, and represents the rate at which fluid elements are deformed:

$$\boldsymbol{D} = \frac{1}{2}(\nabla\boldsymbol{v} + \nabla\boldsymbol{v}^T) \quad (3)$$

Currently, *GOMA* can solve problems for viscoelastic and generalized Newtonian fluids of the Carreau or power law variety. For discussion purposes here we define the stress as $\boldsymbol{T} = \mu\boldsymbol{D} - p\boldsymbol{I}$.

The body force vector, $\boldsymbol{g}$, represents forces acting at a distance on the fluid due to gravity or other sources. A Boussinesq model is available to approximate the effect of buoyancy on a flow field for both continuous and porous media.

One way to model fluid flow in a porous medium amounts to a direct extension of Eq.1 (with limitations laid out in the paper by Gartling et al. 1996). If one considers the porous region to be rigid, isotropic and saturated with an incompressible fluid, this equation can be furbished to model the porous flow. That is, the momentum equations can be augmented for flow in a rigid porous medium using the Brinkman approach. This is turned on in *GOMA* using the POROUS term multiplier and setting the Brinkman-equation parameters in the materials file in a *GOMA* run. This changes several terms in the momentum equations and leads to the so-called Brinkman equation which looks like (Gartling et al. 1996):

$$\frac{\rho}{\phi}\frac{d\boldsymbol{v}}{dt} = -\frac{\rho}{\phi^2}(\boldsymbol{v} - \boldsymbol{v}_m) \bullet \nabla \boldsymbol{v} + \nabla \bullet \boldsymbol{T}_B + \boldsymbol{g} + \left(\frac{\rho \hat{c}}{\sqrt{k}} \|\boldsymbol{v}\| + \frac{\mu}{k} \right) \boldsymbol{v} \quad (4)$$

ϕ is the porosity of the porous medium, $\boldsymbol{T}_B$ is the stress tensor using a Brinkman viscosity (μ_B), $\hat{c}$ is an inertia coefficient, μ is the flowing liquid viscosity, and k is the permeability. Noteworthy here is that the **Viscosity** card in a typical *GOMA* material file corresponds to that which determines the stress T_B term, whereas the **Flowing Liquid Viscosity** card is now needed for the Brinkman term (last term on the right of Eq. 4). It is also important to note that $\boldsymbol{v}$, the velocity, in a porous media becomes a volume-averaged quantity. We must still enforce incompressibility with Eq. 2 so that the pressure in T_B is determinate. Actually Eqs. 2 and 4 represent a significant generalization of the celebrated Darcy model for flow in a saturated porous medium, which can be written in its simplest form as

$$-\nabla p = \frac{\mu}{k}\boldsymbol{v} \quad (5)$$

You can see that this equation can be obtained from Eq. 4 with the porous term (last term on right), and the isotropic portion of the stress tensor T_B. In essence, the so-called Brinkman and Darcy formulations can both be invoked in GOMA using the **POROUS_BRINKMAN** medium model (see **Media Type** card in GOMA manual) by simply setting the equation term multipliers in the *GOMA* input deck and choosing the appropriate properties. To invoke the Brinkman equations in *GOMA* follow these steps

1. Turn on the POROUS multiplier term in the momentum equations, i.e., **momentum1** and **momentum2** equations as described in the *GOMA* manual.

2. Change the media type card from **CONTINUOUS** to **POROUS_BRINKMAN** in the corresponding material file. That is, in the material file that corresponds to the material number of the porous region.

3. Set the porosity, permeability, flowing-liquid viscosity, and inertia coefficient cards in the same material file.

Interestingly, you can turn the Brinkman equations into the Darcy velocity equation with the following procedure:

1. Specify **momentum1**, **momentum2**, and **continuity** equations in the input file for this material

2. Set the DIFFUSION and the POROUS term multipliers in the momentum equations to unity, and all others to zero in the GOMA input file

3. Set the medium type to **POROUS_BRINKMAN** in the material file

4. Set the viscosity value on the regular “**Viscosity**” card to zero and the inertia coefficient to zero.

It is important to retain the continuity equation so that you have an equation for the pressure and velocity components. This is in fact the approach taken by Gartling et al. (1996). It has several advantages over the traditional Darcy potential equation approach discussed below from the standpoint of boundary condition application.

The Brinkman and Brinkman/Darcy approaches to porous media problems, however, currently have some major limitations in GOMA:

1. Cannot connect microstructure changes in properties, like porosity, with the deformation mechanics of the medium. Specifically, the porosity is currently taken as constant in the Brinkman equations and the effective stress principle is not applied to the elastic stress term. This problem could be fixed without too much work.
2. Cannot apply a Lagrangian formulation to the Brinkman equations (see Cairncross et al. 1996). The implication of this is that you cannot do a moving substrate penetration problem, in a fixed laboratory frame. We recommend the approach outlined below for this.
3. The appropriate penetration conditions for a Lagrangian mesh motion type in the porous medium do not exist. Actually, this is related to the lack of Lagrangian terms in the stress equation for the porous network. These terms are not added for a **`POROUS_BRINKMAN`** media type.

These limitations do not preclude coupled continuous fluid/porous media problems, with mass exchange between the media. Subject to the limitations outlined in the paper by Gartling, et al. (1996), GOMA can currently perform these problems as long as the porous structure is rigid, isotropic and fixed in the computational frame of reference.

The Darcy equation can be invoked another way within GOMA using the species equations. It is through this approach that one can currently invoke the physics required by Lagrangian poroelastic stress problems for both saturated and unsaturated media. Even though we focus in this tutorial on the former, we will state the equations in the general unsaturated form.

For problems involving partially-saturated flow in porous media, the conservation equations for solvent and air are derived for flow and diffusion in both the liquid and gas phases (Martinez, 1995):

$$\frac{dC_i}{dt} = (\boldsymbol{v}_m + \boldsymbol{v}_s) \bullet \nabla C_i - \nabla \bullet [\boldsymbol{v}_s C_i + \boldsymbol{v}_g \rho_{gi} + \boldsymbol{v}_l \rho_{li} + \boldsymbol{J}_{gi} + \boldsymbol{J}_{li}] \tag{6}$$

Here, C_i is the total concentration of species i from both phases (per unit volume of media). Currently *GOMA* is set up for *unsaturated* or *two-phase* transport in porous media. In unsaturated flow, the liquid phase is pure solvent, and the gas phase is assumed to be at a constant pressure; species equation one (i=1) corresponds to the mass balance for the solvent and the species variable is the capillary pressure. In two-phase flow, the liquid phase is pure solvent, and the gas phase contains both air and solvent vapor; species equation one (i=1) corresponds to the mass balance for the solvent, and species equation two corresponds to the mass balance for air. ν_g and ν_l are the velocities of gas and liquid relative to the solid skeleton and are normally calculated from Darcy's Law. ρ_{gi} and ρ_{li} are the concentrations of species i in the gas and liquid phases, and should be in local equilibrium. J_{gi} and J_{li} are the diffusion fluxes of species i in the gas and liquid phases. In *GOMA* the whole last term is considered the diffusion term, even though it includes convection in the gas and liquid phases. If the porosity is an unknown (deformable porous media), it is represented by the variable for species two in unsaturated media and species three in two-phase media using equation:

$$det(F) = (1 - \phi_0) / (1 - \phi) \tag{7}$$

Here, ϕ_0 is the porosity of the stress free state and $\boldsymbol{F}$ is the deformation gradient tensor (see Cairncross et al. for details). It turns out that GOMA uses the second species equation to accommodate this kinematic constraint, with the advective term only. For the remainder of this tutorial, we will focus on the single component, saturated case, with the intent on instructing the user how to analyze a porous-penetration problem in moving/deforming medium. It is the author's experience that unsaturated problems are much harder to solve. A memo addressing the unsaturated part is forthcoming (GT-009.3).

For problems involving saturated flow of liquid in porous media, liquid flows by Darcy's Law:

$$\frac{d\phi}{dt} = (\boldsymbol{v}_m + \boldsymbol{v}_s) \bullet \nabla\phi - \nabla \bullet \left[\boldsymbol{v}_s \phi + \frac{k}{\mu} \nabla p_c \right] \tag{8}$$

Here, ϕ is the porosity of the porous medium. Currently *GOMA* is set up so that species equation one (i=1) corresponds to flow of solvent using the species variable to represent the capillary pressure, p_c. As pointed out above, if the porosity is an unknown (deformable porous media), it is represented by the variable for species 2 using Equation 7. The first term on the left, and the first term on the right account for changing porosity with time and space. Notice how the term involving $\boldsymbol{v}_s \bullet \nabla\phi$ cancels from this equation if the chain rule of differentiation is invoked. This implies that flux of the fluid due to motion of the porous network is taken up in the Lagrangian mesh velocity term ν_m. Equation 8 is derived by substituting Equation 5, with an additional convective flux term into a generalized version of Equation 2 that accounts for the deformation of the medium. This is analogous to the Eulerian form of the continuity equation for a compressible medium, viz.

$$\frac{d\phi}{dt} = (\boldsymbol{v}_m + \boldsymbol{v}_s) \bullet \nabla\phi - \nabla\bullet[v] \qquad (9)$$

In the case of a rigid medium with no advected Lagrangian velocity, Equation 8 becomes a Laplace equation for the pressure of the pore fluid.

In order to apply the Darcy-potential form to a porous penetration problem, several important boundary conditions are needed. Here, for the sake of completeness, we present all of the boundary conditions currently supported in GOMA that pertain to the Darcy-potential formulation:

•Boundary Conditions for Darcy Potential Formulation (Equation 8)

GOMA BC Name and Description	Mathematical Form and Differential Equation to which it is applied	Formulation
DARCY_CONTINUOUS Applied to a side set. Continuity of velocity at an interface between a continuous fluid and a porous medium.	$\underset{\sim}{n} \cdot (\underset{\sim}{v} - \underset{\sim}{v}_m) = -k/\mu\nabla p_c$ [a] Applied to normal component of continuous fluid momentum equation at the interface.	Darcy potential Form. Saturated
POROUS_PRESSURE Applied to a node set. Set the continuity of hydrodynamic pressure in the continuous fluid to the Darcy pressure in the porous medium, at the interface.	$p = p_c$ Applied to the species 0 equation from the porous medium at the interface. i.e. $p = y_0$	Darcy potential Form Saturated
VELO_TANGENT_SOLID Applied to a side set. Set tangential fluid velocity to the tangent velocity of an advecting/deforming porous medium	$t \bullet v = t \bullet v_s$ Equivalent to tangential component of the NO_SLIP condition (see Schunk et al. 1998) but allows for penetration. Applied to tangential component of fluid momentum equation from continuous phase.	Any form. Saturated or unsaturated.
POROUS_KIN Applied to a side set. Set mass loss rate in a single phase deformable porous medium problem equivalent to the surface recession rate times a density.	$\rho\underset{\sim}{n} \cdot \underset{\sim}{v}_m = -(\rho k)/\mu\nabla p_c$ Applied to the normal component of the Lagrangian mesh equations. This is a distinguishing condition for an imbibing or expelling porous deformable medium.	Darcy-potential form only. Single phase Saturated
VP_EQUIL Applied to a side set. Vapor-liquid equilibrium for adjacent porous medium and fluid (gas) phases.	Not recently tested. Looks specialized for a gas overlying a vapor in a porous phase.	Darcy - potential form. Unsaturated

•Boundary Conditions for Darcy Potential Formulation (Equation 8)

GOMA BC Name and Description	Mathematical Form and Differential Equation to which it is applied	Formulation
VN_POROUS Applied to a side set. The unsaturated equivalent to DARCY_CONTINOUS. Includes Darcy flux and Fickian diffusive flux in porous phase.	$n \cdot (v_g \rho_{g_i} + v_l \rho_{l_i} + J_{g_i} + J_{l_i}) = n \cdot (v - v_m)$	Darcy-potential form. Unsaturated.
POROUS_CONV Applied to a side set. Calculates normal flux of material at an interface of a Lagrangian and continuous medium	Not recently tested.	Darcy potential Form.
POROUS_FLUX Applied to a side-set Set flux of solvent at surface of porous media equal to mass transfer coefficient times driving force	$n \cdot (v_g \rho_{g_i} + v_l \rho_{l_i} + J_{g_i} + J_{l_i}) = h\phi(\rho_{g_i} - \rho^0_{g_i})$ Untested since ~1996.	Darcy-Potential form. Unsaturated

a. p_c is taken as the first species, accounted for by a species convective-diffusion equation, in this formulation

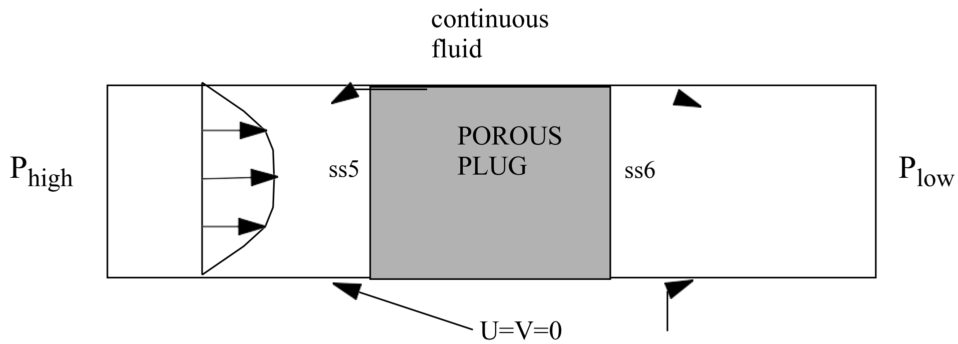
The current limitations of the Darcy-Potential approach, which we apply below to a porous penetration problem on a moving web, are as follows:

1. No velocity components are computed in the porous phase. They must be backed-out from a user-defined post processing variable.
2. Although tested thoroughly for a saturated case, there are still some convergence problems at large deformations, and with the **DEFORM** porosity model.
3. Jacobian errors abound in the Darcy equation; the errors appear inconsequential to the convergence.
4. Unchecked form of boundary conditions. Need to check whether porosity multipliers are needed a'la Scherer et al. for the porous pressure at the interface between a porous medium and a continuous phase. Also need to add the normal viscous stress piece from the fluid in POROUS_PRESSURE.

With these equations and boundary conditions, we now study two instructional problems. The first is the well-known porous-plug problem which demonstrates three approaches to solving a porous penetration problem in a rigid porous medium. The second is a generalized knife coating problem covered in the GOMA Beginner's tutorial, but with the additional complication of a moving, deformable porous substrate.

The porous plug problem. Saturated and Rigid

This problem tests several coupled porous flow formulations and the accompanying boundary conditions between porous and fluid regions (files are in subdirectory **general_plug**). The problem is diagrammed below



This problem can be solved with GOMA in several ways. The following problem description section must be used for a Brinkman solution (all this is driven by the input file **`brinkman_input`** in the template directory):

```
Number of Materials = 3

MAT = fluid1 1
    Coordinate System = CARTESIAN
    Element Mapping = isoparametric
    Mesh Motion = ARBITRARY
    Number of bulk species = 0
    Number of EQ = 3
    EQ = momentum1 Q2   U1 Q2   0.  1.  1. 1. 0.   0.
    EQ = momentum2 Q2   U2 Q2   0.  1.  1. 1. 0.   0.
    EQ = continuity Q1 P   Q1 1.   0.
                  div ms adv bnd dif src por

MAT = substrate_brink 2
    Coordinate System = CARTESIAN
    Element Mapping = isoparametric
    Mesh Motion = ARBITRARY
    Number of bulk species = 1
    Number of EQ = 3
    EQ = momentum1 Q2   U1 Q2   0.  1.  1. 1.   0.  1.
    EQ = momentum2 Q2   U2 Q2   0.  1.  1. 1.   0.  1.
    EQ = continuity Q1 P   Q1 1.   0.

MAT = fluid2 3
    Coordinate System = CARTESIAN
    Element Mapping = isoparametric
    Mesh Motion = ARBITRARY
    Number of bulk species = 0
    Number of EQ = 3
    EQ = momentum1 Q2   U1 Q2   0.  1.  1. 1.   0.  0.
    EQ = momentum2 Q2   U2 Q2   0.  1.  1. 1.   0.  0.
    EQ = continuity Q1 P   Q1 1.   0.
```

There are three materials: the first and third are continuous fluid media with the same properties and the second region (in the middle) is a porous plug. In the continuous fluid regions, whose corresponding **mat** files are called **fluid1.mat** and **fluid2.mat**, we are solving the standard fluid momentum equations. Those material files specify a Newtonian, incompressible material so that these equations are just the Navier Stokes equations. In the porous plug, it appears we still are solving what looks like the Navier-Stokes equations, except that the porous term multiplier (the last multiplier on **momentum1** and **momentum2**) is turned to unity. This invokes the last term on the right in Equation 4, which is the so-called Brinkman term. In the corresponding material file, called **substrate_brink.mat,** we have the following cards under the microstructure properties section:

```
---Microstructure Properties
Media Type              = POROUS_BRINKMAN
Porosity                = CONSTANT    0.2
Permeability            = CONSTANT1.0e-5
FlowingLiquid Viscosity = CONSTANT 10.
Inertia Coefficient     = CONSTANT 0.0
```

Notice that the **Media Type** card is set to **POROUS_BRINKMAN**. The four physical properties required by Equation 4, in addition to the density and the regular viscosity, are specified on the four following cards. *RECALL again that the viscosity used to determine the Brinkman stress term T_B is specified through the Viscosity which is a 'Mechanical Properties and Constitutive Equation' parameter*. The results of running this problem agree with the solutions presented by Gartling, et al. (1996), with the same physical parameters. Consult that paper if you are interested in details.

You can determine the pressure drop in this porous material with a Darcy approach using the same specifications, simply by turning off the advective term and setting the viscosity (NOT the Flowing Liquid Viscosity) to zero. Equation 4, together with Equation 2 comprise the Darcy formulation, i.e., Equation 5, without inertia, the pressure gradient coming from the T_B term. This approach is convenient as the results contain both the pressure and the velocity components in the porous medium. The boundary conditions in both of these approaches are the same, with no special considerations at the interface. The accuracy of the naturally arising continuity in velocity that results at the interface can be debated (see Gartling, et al. for a discussion), but nonetheless a reasonable solution results. You can run this test case in the same template directory by running the file **darcy_input**.

It is perhaps most instructive in this problem to see what must be done in order to solve for just the porous pressure in the plug with a Darcy-potential approach. First, the problem description for the plug material changes, viz.

```
MAT = substrate 2
    Coordinate System        = CARTESIAN
    Element Mapping          = isoparametric
    Mesh Motion              = ARBITRARY
    Number of bulk species   = 1

    Number of EQ             = 1
```

EQ = species_bulk Q1 Y Q1 0. 1. 1. 1. 1. 1.

The potential equation, or the equation that arises when Equation 5 is substituted into Equation 2, is invoked through the bulk species equation. With the mass, advective and diffusive terms on, the equation looks like Equation 8 above. The first species (species 0), is the pressure in the porous medium. If the medium is deformable, a second species equation is invoked to account for the porosity. This tutorial does not address this complication. Notice in this specification that there is one bulk species and also that the interpolation and weighting function are set to linear Q1. The underlying reason for this choice will become apparent below.

Special boundary conditions are required to solve the problem with the Darcy-potential equation that are designed to connect the pressure gradient at the interface, on the side of the porous media, with the normal component of velocity on the fluid side. The **DARCY_CONTINUOUS** condition outlined in the table above was added to GOMA for this case. Because this is a saturated, rigid case, we do not require the more sophisticated **VN_POROUS** condition. In addition to this statement of mass conservation, we also need a statement of continuity between the pore liquid pressure p_c and the continuous hydrodynamic pressure p. The **POROUS_PRESSURE** condition described in the table above serves as this "tie" constraint. Were the pressure variable slots in GOMA the same on either side of the interface, continuity would occur naturally as the pressure would be shared at the interface. However, because we use the bulk species, the pore pressure is actually held in the "concentration" variable slot, necessitating such a condition. This is further complicated however if you are carrying a concentration variable in the liquid phases, as the interfacial conditions then require the use of a jump condition. This will be taken up in a future memo. Of course we need these on both sides of the plug, as is demonstrated in the boundary condition section of the input file, which is called **potential_input**.

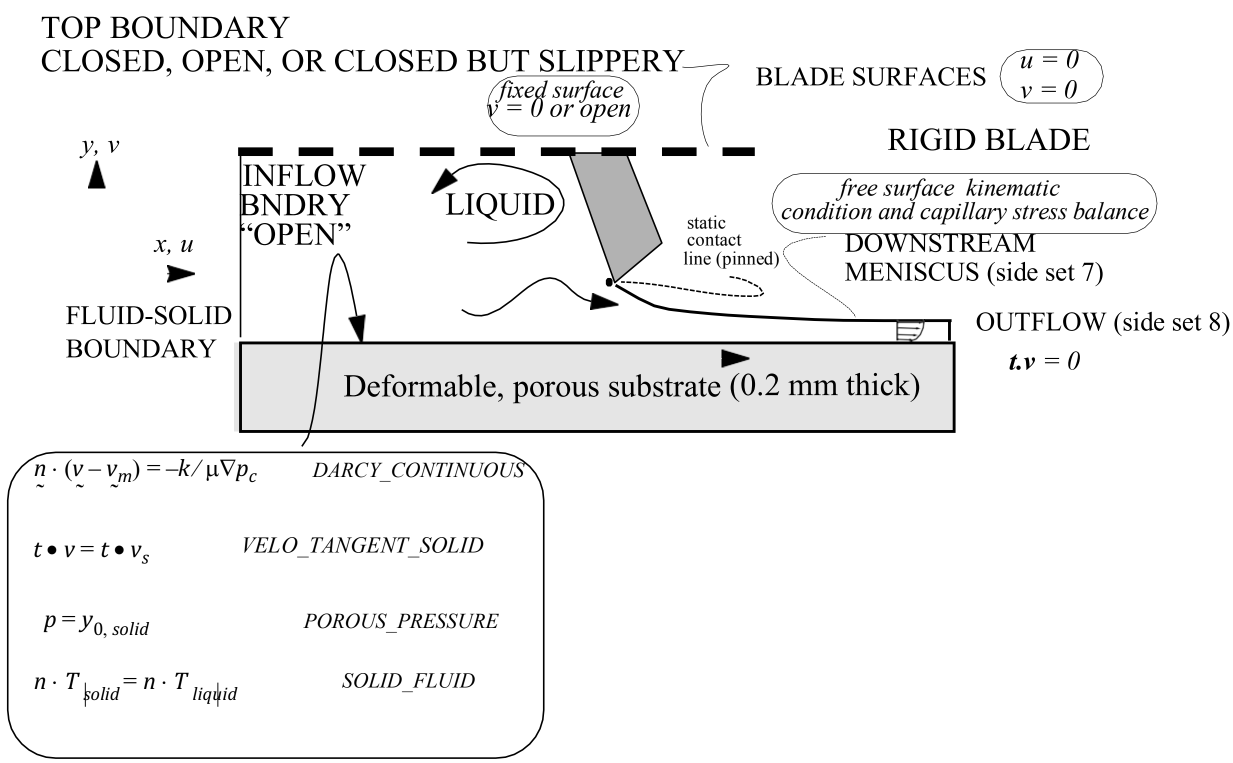
```
------------------------------------------------------------
    Boundary Condition Specifications
------------------------------------------------------------
Number of BC = -1
$$

BC     = POROUS_PRESSURE SS 5 2 1
BC     = POROUS_PRESSURE SS 6 2 3
                                           ^ Solid porous phase material ID
                                             ^Liquid phase material IDs.

BC     = DARCY_CONTINUOUS SS 5 2 1
BC     = DARCY_CONTINUOUS SS 6 2 3

$$ side wall top
BC     = U NS 2 0
BC     = V NS 2 0

$$side wall bottom
BC     = U NS 4 0.
BC     = V NS 4 0.
```

```
$$inflow
BC     = U NS 3 0.
BC     = FLOW_PRESSURE SS 3 10.

$$outflow
BC     = FLOW_PRESSURE SS 3 -10

END OF BC
```

A sample solution is shown on the following page.

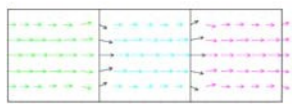
Coating on a porous, deformable substrate

To illustrate the application of the Darcy-potential approach to modeling a coating penetration problem, the knife coating template detailed in the beginners tutorial (GT-001.4, contact author) has been augmented with a substrate material that is porous and deformable (files are in subdirectory **knife**). The problem geometry and relevant boundary conditions are illustrated in the figure below.

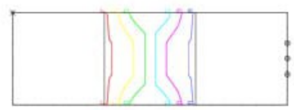


A detailed description of the relevant fluid phase boundary conditions can be found in the beginner's tutorial. The most important ones in that phase are the kinematic boundary condition, the capillary stress balance applied on the free surface (side set #7) and the fully-developed flow conditions applied to the outflow plane (side set #8). The liquid properties are those of a Newtonian liquid, and can be found in the material file `coating_liq.mat`.

Brinkman solution. brinkman_input

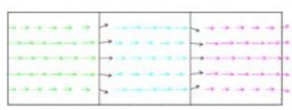


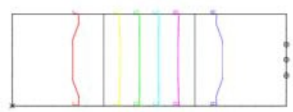
velocity



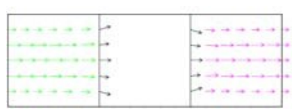
pressure

Darcy solution using only Brinkman term. darcy_input

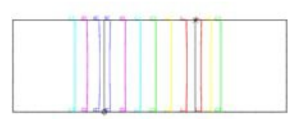




Darcy-potential solution. potential_input



pressure



y0, pore pressure

To extend this tutorial problem to the impregnation case, we added a second material region (Region #4 in the **knife.fas** file). That region was given a material name of 2, with the liquid region being material 1. The GOMA input file, **knife_input**, was then augmented as follows (only the boundary conditions and problem description sections are shown here):

```
.......

{delta_gap = Gap - 0.0001}

Number of BC = -1
BC = PLANE SS 2 {a2} {b2} {c2} {d2}
BC = PLANE SS 3 {a3} {b3} {c3} {d3}
BC = PLANE SS 4 0. 1. 0.-{H}
BC = PLANE SS 5 {a5} {b5} {c5} {d5}
BC = PLANE SS 6 {a6} {b6} {c6} {d6}
BC = PLANE SS 8 1. 0. 0. {-S}
BC =    DX NS 1002 0
BC =    DY NS 1002 {delta_gap} 1.0
BC =    DX NS 1001 0
BC =    DY NS 1001 {delta_gap} 1.0
BC =    DX NS 1000 0
BC = DY NS 1000 {delta_gap} 1.0

BC = VELO_TANGENT_SOLID SS 2 2 1
BC = SOLID_FLUID SS 2 2 1
$$
BC = POROUS_PRESSURE SS 2 2 1
BC = DARCY_CONTINUOUS SS 2 2 1

BC =    V NS 3 0.

BC =    V NS 4 0.

BC =    U NS 5 0.
BC =    V NS 5 0.

BC =    U NS 6 0.
BC =    V NS 6 0.

$$BC = VELO_NORMAL SS 7 0.
BC = KINEMATIC SS 7 0.
BC = CAPILLARY SS 7 1.01 0. 0. 0.
BC = SURFTANG NS 200 1. 0. 0. -1.01

BC = VELO_TANGENT SS 8 0 0. 0. 0.
BC = V NS 8 0.

BC =    DX NS 100 {x32_new - x32} 1.0
BC =    DY NS 100 {y32_new - y32} 1.0

$$ Backpressure on underside of web (see page 109 of 2.0 Revised edition man)
BC =    Y NS 1001 0 0.
END OF BC

###########
----
Problem Description
---

Number of Materials = 2
```

```
MAT = coating_liq 1
      Coordinate System = CARTESIAN
      Element  Mapping  =  isoparametric
      Mesh    Motion    =    ARBITRARY
      Number of bulk species = 0

      Number of EQ = 5
          EQ = mesh1          Q2    D1    Q2       0.  0.  0.  1.   0.  0.
          EQ = mesh2          Q2    D2    Q2       0.  0.  0.  1.   0.  0.
          EQ = momentum1      Q2    U1    Q2       0.  1.  1.  1.   1.  0.
          EQ = momentum2      Q2    U2    Q2       0.  1.  1.  1.   1.  0.
          EQ = continuity     Q1    P     Q1    1.               0.

MAT = substrate 2
      Coordinate System = CARTESIAN
      Element Mapping = isoparametric
      Mesh Motion       = LAGRANGIAN
      Number of bulk species = 1

      Number of EQ = 3
          EQ = mesh1          Q2    D1    Q2       0.   0.   0. 1.   0. 0.
          EQ = mesh2          Q2    D2    Q2       0.   0.   0. 1.   0. 0.
          EQ = species_bulk   Q1    Y     Q1       0.   1.   1. 1.   1. 1.
```

The lines in bold type in the **knife_input** excerpt are the focus of this discussion. All others are discussed in detail in the knife coating tutorial. First, with regard to the problem description section, it is clear that in each material we require the mesh motion equations, viz., **mesh1** and **mesh2**. In the substrate material (#2) the mesh motion is set to **LAGRANGIAN**. Correspondingly, in the **substrate.mat** file we specify a stress-free-state motion with the **Advected Lagrangian Velocity** card. The tangential velocity component of the liquid along the substrate surface, or side set #2, is enforced through the **VELO_TANGENT_SOLID** boundary condition. If the substrate were impenetrable, this boundary condition would be exchanged for the **NO_SLIP** boundary condition, which really sets the velocity components of the fluid to match those of the advecting and deforming solid, without mass exchange.

Transport of pore fluid in the substrate is accounted for with the species bulk equation. Several features of that equation are noteworthy. First, the number of bulk species is set to 1, which implies that we are not tracking a variable porosity in this example. This also means that the pore pressure can be viewed in the output **EXODUSII** file as the **Y0** variable. The most important feature of this card is that we use a **Q1** interpolation for the **Y** variable (i.e., the pore pressure). This is necessitated by the **Q1** interpolation of the hydrodynamic pressure in the adjacent liquid phase, accounted for as usual by the continuity equation. All this comes about because of the well-known LBB condition on the allowable choices of pressure interpolation with an finite element formulation of the Navier-Stokes equations. The upshot is that we must use an interpolation and weighting polynomial that are one order lower than those used for the velocity, which in this case is quadratic (Q2). This is where the POROUS_PRESSURE boundary condition plays a role in setting the only allowable choice of interpolation for the species (or pore pressure): since it is responsible for connecting the bulk fluid pressure with the pore pressure, this boundary conditions forces us to represent the species with the same order of interpolation as that used for the bulk pressure, or **Q1**.

Connecting the pressure between the phases is not sufficient to complete the problem. For one thing, that condition alone may be inaccurate as some argue that the liquid phase pressure and the normal viscous stress should be pitted against the pore pressure (See Scherer 1996). However, the debate continues and sound engineering judgement must be brought into play in order make use of the results. Also, we must say something about mass conservation between phases. The Navier-Stokes equations in the fluid phase require at least 3 boundary conditions: one on pressure, and one on each of the components of velocity. **VELO_TANGENT_SOLID** and **POROUS_PRESSURE** provide two conditions, and the third must account for the normal component of velocity.

DARCY_CONTINUOUS is a boundary condition that equates the velocity component in the liquid phase normal to the interface with the Darcy velocity, normal to the same interface. Again, whether there should be an adjustment for the effective “pore area” can be debated. Nonetheless with this condition, the mass conservation constraints are now complete.

One last condition required at the interface is a balance of stress to help determine additional unknowns there, i.e., the coordinate values of the interface position. The **SOLID_FLUID** boundary condition is a vector condition which takes care of this.

Starting up this problem should be pursued in the same way as is described in the Beginner’s tutorial memo. Here we will add on some suggestions which apply to the deforming substrate, with the penetration.

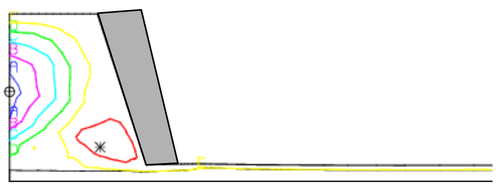
First, start with a “rigid” substrate. Set the Lame coefficients in **substrate.mat** to large numbers (> 1.e7). Once you have a solution, you can soften the web and decrease the gap. A sample solution is shown below. In this solution it is clear that the high metering pressure causes penetration upstream of the knife, and the low pressure that results due to the expansion after the metering blade causes the substrate to expel liquid.

In summary:

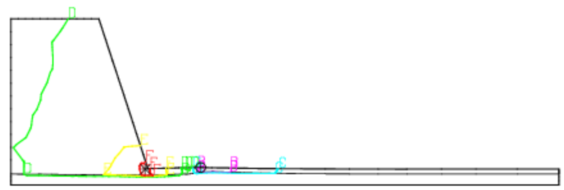
KEY POINTS WHEN RUNNING PENETRATION PROBLEMS:

1. If the solid body is undergoing user-prescribed motion, and/or is deformable, you must use the Darcy-potential formulation to track penetration.
2. To connect the pore pressure, or the species 0 field, in the solid to the adjacent continuous fluid you need to apply the POROUS_PRESSURE boundary condition. This forces the interpolation functions for the species 0 in the solid porous phase to be set equal to that used for the pressure (on the continuity equation) in the adjacent bulk phase, subject to all the constraints on it.

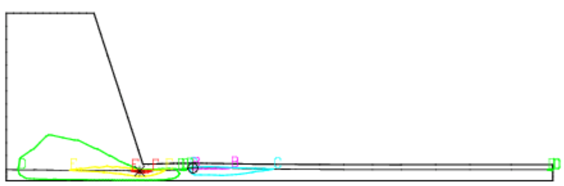
pattern of streamlines



liquid pressure



pore pressure



3. Only saturated penetration has been tested at this point, although the full compliment of conditions and equations are available for deformable, unsaturated media.

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